General relation between quantum ergodicity and fidelity of quantum dynamics

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A general relation is derived, which expresses the fidelity of quantum dynamics, measuring the stability of time evolution to small static variation in the Hamiltonian, in terms of ergodicity of an observable generating the perturbation as defined by its time correlation function. Fidelity for *ergodic* dynamics is predicted to decay *exponentially* on time scale $\propto\delta^{-2}$, $\delta\sim$ strength of perturbation, whereas faster, typically *Gaussian* decay on shorter time scale $\propto\delta^{-1}$ is predicted for *integrable*, or generally *nonergodic* dynamics. This result needs the perturbation δ to be sufficiently small such that the fidelity decay time scale is larger than any (quantum) relaxation time, e.g., mixing time for mixing dynamics, or averaging time for nonergodic dynamics (or Ehrenfest time for wave packets in systems with chaotic classical limit). Our surprising predictions are demonstrated in a quantum Ising spin-(1/2) chain periodically *kicked* with a tilted magnetic field where we find finite parameter-space regions of nonergodic and nonintegrable motion in the thermodynamic limit.

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The quantum signatures of various types of classical motion, ranging from integrable to ergodic, mixing and chaotic, are still lively debated issues (see, e.g., Ref. [1]). Most controversial is the absence of exponential sensitivity to variation of initial condition in quantum mechanics, which prevents direct definition of quantum chaos [2]. However, there is an alternative concept that can be used in classical as well as in quantum mechanics [3]: One can study the stability of motion with respect to small variation in the Hamiltonian. Clearly, in classical mechanics this concept, when applied to individual trajectories, is equivalent to sensitivity to initial conditions. Integrable systems with regular orbits are stable against small variation in the Hamiltonian (the statement of KAM theorem), wheres for chaotic orbits varying the Hamiltonian has similar effect as varying the initial condition: exponential divergence of two orbits for two nearby chaotic Hamiltonians.

The quantity of the central interest here is the *fidelity* of quantum motion. Consider a unitary operator U being either (i) a short-time propagator, or (ii) a Floquet map $U=\hat{\mathcal{T}}\exp[-i\int_0^p d\tau H(\tau)/\hbar]$ of (periodically time-dependent) Hamiltonian $H[H(\tau+p)=H(\tau)]$, or (iii) a quantum Poincaré map. The influence of a small perturbation to the unitary evolution, which is generated by a Hermitian operator A, $U_\delta=U\exp(-iA\delta)$, δ being a small parameter, is described by the overlap $\langle\psi_\delta(t)|\psi(t)\rangle$ measuring the Hilbert space distance between exact and perturbed time evolution from the same initial pure state $|\psi(t)\rangle=U^t|\psi\rangle, |\psi_\delta(t)\rangle=U_\delta^t|\psi\rangle$, where *integer* t is a discrete time (in units of the period p) [4]. This defines the *fidelity*

$$F(t)=\langle U_\delta^{-t}U^t\rangle, \tag{1}$$

where the average is performed either over a fixed pure state $\langle\cdot\rangle=\langle\psi|\cdot|\psi\rangle$, or, if convenient, as a uniform average over all possible initial states $\langle\cdot\rangle=(1/\mathcal{N})\mathrm{tr}(\cdot)$, $\mathcal{N}$ being the Hilbert space dimension. The quantity $F(t)$ has already raised considerable interest, though under different names and interpretations: First, it has been proposed by Peres [3] as a measure of stability of quantum motion. Second, it is the *Loschmidt echo* measuring the *dynamical irreversibility of quantum phases*, used, e.g., in spin-echo experiments [5] where one is interested in the overlap between the initial state $|\psi\rangle$ and a state $U_\delta^{-t}U^t|\psi\rangle$ obtained by composing forward time evolution, imperfect time inversion with a residual interaction described by the operator $A\delta$, and backward time evolution. Third, the fidelity has become a standard measure characterizing the loss of phase coherence in quantum computation [6]. Fourth, it was used to characterize ''hypersensitivity to perturbation'' in related studies [7], though in different contexts of stochastically time-dependent perturbation.

The main result of this paper is a relation of the fidelity to ergodic properties of quantum dynamics, more precisely to the time autocorrelation function of the generator of the perturbation A. Quantum dynamics of finite and bound systems has always a *discrete spectrum* since the effective Hilbert space dimension $\mathcal{N}$ is finite, hence it is *nonergodic* and *nonmixing* [8,9]: time correlation functions have fluctuating tails of order $\sim 1/\mathcal{N}$. In order to reach genuine complexity of quantum motion with possibly continuous spectrum one has to enforce $\mathcal{N}\to\infty$ by considering one of the following two limits: quasiclassical limit of effective Planck's constant $\hbar\to 0$, or thermodynamic limit (TL) of number of particles, or size $L\to\infty$. Our result is surprising in the sense that it predicts the *average* fidelity to exhibit exponential decay on a time scale $\propto\delta^{-2}$ for *ergodic systems* (i.e., such that the integrated time autocorrelation of A is finite), but much faster, typically Gaussian decay on a shorter time scale $\propto\delta^{-1}$ for integrable and general nonergodic systems (i.e., such that time averaged autocorrelation of A is nonvanishing). Our theory on fidelity is very general and can be extended to any perturbed unitary evolution, either in quantum, quasiclassical, or even classical (Liouvillian) context. In this paper we apply it to the *quantum many-body* problem in TL, in particular in the *kicked Ising model* (KI), namely, the Ising spin-(1/2) chain periodically kicked with a tilted homogeneous magnetic field. KI is particularly interesting since it possesses parameter-space regions with positive measure of *nonergodic* behavior in TL surrounding the integrable cases [10] of vanishing measure, which is an additional evidence for a conjecture [9] on existence of intermediate, nonintegrable and nonergodic quantum motion of disorderless interacting many-body systems in TL.

We start by rewriting the fidelity (1) in terms of a Heisenberg evolution of the perturbation $A_t := U^{-t} A U^t$

$$F(t) = \langle e^{iA_0\delta} e^{iA_1\delta} \cdots e^{iA_{t-1}\delta} \rangle = \hat{\mathcal{T}} \left\langle \prod_{t'=0}^{t-1} \exp(iA_{t'}\delta) \right\rangle \tag{2}$$

which is achieved by t insertions of the unity $U^{-t'}U^{t'}$ and recognizing $U^{-(t'-1)} U_\delta^\dagger U^{t'} = \exp(i\delta A_{t'-1})$. $\hat{\mathcal{T}}$ is a left-to-right time ordering. Next we make an expansion in δ expressing the fidelity in terms of correlation functions

$$F(t) = 1 + \sum_{m=1}^{\infty} \frac{i^m \delta^m}{m!} \hat{\mathcal{T}} \sum_{t_1, t_2 \ldots t_m = 0}^{t-1} \langle A_{t_1} A_{t_2} \cdots A_{t_m} \rangle. \tag{3}$$

Being interested mainly in the absolute value $|F(t)|$, we will, in the following, choose perturbations with vanishing first moment $a := (1/t)\Sigma_{t'=0}^{t-1} \langle A_{t'} \rangle = 0$ so that the series (3) starts at $m=2$, since a shift by a multiple of unity $A \to A - a1$ simply rotates the fidelity $F(t) \to \exp(-ia\delta) F(t)$. To second order in δ we have

$$F(t) = 1 - \frac{\delta^2}{2} \sum_{t'=-t}^{t} (t - |t'|) C_A(t') + \mathcal{O}(\delta^3), \tag{4}$$

where it is assumed that two-point time correlation function is homogeneous $C_A(t'-t) := \langle A_t A_{t'} \rangle$, as is the case for uniform average over initial states $\langle \cdot \rangle = \mathrm{tr}(\cdot)/\mathcal{N}$. Equation (4) reveals a simple general rule: the stronger the correlation decay, the slower is the decay in fidelity, and vice versa. Below we discuss two different cases in the limit $\mathcal{N} \to \infty$.

(I) Ergodicity and fast mixing. Here we assume that $C_A(t) \to 0$ sufficiently fast that the total sum converges, $S_A := (1/2)\Sigma_{t=-\infty}^{\infty} C_A(t), |S_A| < \infty$. For times t much larger than the so-called *mixing time scale* $t \gg t_{\mathrm{mix}}$, which effectively characterizes the correlation decay, e.g., $t_{\mathrm{mix}} = \Sigma_t |t C_A(t)| / \Sigma_t |C_A(t)|$, it follows that the fidelity drops linearly in time $F_{\mathrm{e}}(t) = 1 - t/\tau_{\mathrm{e}} + \mathcal{O}(\delta^3)$ on a scale

$$\tau_{\mathrm{e}} = S_A^{-1} \delta^{-2}. \tag{5}$$

In order to show even stronger result, we further assume fast mixing with respect to product observables $B_{tt'} = A_t A_{t'}$ with $\langle B_{tt'} \rangle = C_A(t'-t)$, of order $k \geq 2$, namely $\langle B_{t_1 t_2} B_{t_3 t_4} \cdots B_{t_{2k-1} t_{2k}} \rangle \to \Pi_{j=1}^{k} \langle B_{t_{2j-1} t_{2j}} \rangle$ as $t_1, t_2, \ldots$ are ordered and $t_{2j+1} - t_{2j} \to \infty$. Therefore, the leading contribution for large t to each m term of Eq. (3) comes from sequences $(t_1, t_2, \ldots, t_m)$ where consecutive pairs (t_{2j-1}, t_{2j}) are close to each other, $t_{2j} - t_{2j-1} \lesssim t_{\mathrm{mix}}$. Since for odd m time indices cannot be paired these terms should vanish asymptotically (as $t \to \infty$) relatively to even m terms. Thus we can evaluate $(2k-1)!!$ equivalent even $m = 2k$ terms in Eq. (3) as k tuple of independent sums over $t'_j = t_{2j} - t_{2j-1}$ giving, for $t \gg t_{\mathrm{mix}}$,

$$F_{\mathrm{e}}(t) = \sum_{k=0}^{\infty} \frac{(-1)^k (2k-1)!! 2^k \delta^{2k} S_A^k}{(2k)!} = \exp(-t/\tau_{\mathrm{e}}). \tag{6}$$

Note that formulas (5) and (6) remain valid in a more general case of inhomogeneous time correlation where one should take $S_A := \lim_{t\to\infty}(1/t)\Sigma_{t,t'=0}^{\infty} \langle B_{tt'} \rangle$.

(II) Nonergodicity. Here we assume that the autocorrelation function of the perturbation does not decay asymptotically but has a nonvanishing time average, $D_A := \lim_{t\to\infty}(1/t)\Sigma_{t'=0}^{t-1} C_A(t')$, though the first moment is vanishing $\langle A \rangle = 0$. For times t larger than the *averaging time* t_{ave} in which a finite time average effectively relaxes into the stationary value D_A, we can write fidelity to second order, which decays quadratically in time, $F_{\mathrm{ne}}(t) = 1 - (1/2) \times (t/\tau_{\mathrm{ne}})^2 + \mathcal{O}(\delta^3)$, on a scale

$$\tau_{\mathrm{ne}} = D_A^{-1/2} \delta^{-1}. \tag{7}$$

More general result can be formulated in terms of a time-averaged operator $\bar{A} := \lim_{t\to\infty}(1/t)\Sigma_{t'=0}^{t-1} A_{t'}$, namely, for $t \gg t_{\mathrm{ave}}$ Eq. (3) can be rewritten as

$$F_{\mathrm{ne}}(t) = 1 + \sum_{m=2}^{\infty} \frac{i^m \delta^m t^m}{m!} \langle \bar{A}^m \rangle = \langle \exp(i\bar{A}\delta t) \rangle. \tag{8}$$

Global behavior of $F_{\mathrm{ne}}(t)$ for nonergodic systems, where higher m terms of Eq. (3) become important, depends generally on the full sequence of moments $\langle \bar{A}^m \rangle$. We argue below, by giving an example of spin-(1/2) chains, that there are large classes of perturbing operators where these moments can be shown to possess normal Gaussian behavior, yielding Eq. (9). Nonergodic behavior is certainly present for generic observables in *completely integrable systems* where a sequence of conservation laws can be used to estimate the time-averaged correlator D_A [11], but we wish to make a stronger statement, namely, that there is a generic regime of intermediate dynamics in nonintegrable systems displaying nonergodic behavior [9].

Let us now apply our theory to quantum spin-(1/2) chains described by Pauli operators σ_j^{xyz} on a periodic lattice of size $L, j+L \equiv j$, acting on a Hilbert space of dimension $\mathcal{N} = 2^L$, fix the average $\langle \cdot \rangle = \mathrm{tr}(\cdot)/\mathcal{N}$, and assume that our Floquet operator U is *translationally invariant* (TI) on a lattice. It is useful to introduce a set of *local* TI observables $Z_{\underline{s}} = L^{-1/2}\Sigma_j \sigma_j^{s_0} \sigma_{j+1}^{s_1} \cdots \sigma_{j+n}^{s_n}$, of *order* $n \ll L$, where $\underline{s} = [s_0, s_1, \ldots, s_n], s_0, s_n \in \{x,y,z\}, s_j \in \{0,x,y,z\}, 1 \leq j \leq n-1$, and $\sigma_j^0 := 1$. Using $\langle \sigma_j^s \sigma_k^r \rangle = \delta_{j,k} \delta_{s,r}$ one may derive a contraction formula

$$\langle Z_{\underline{s}_1} Z_{\underline{s}_2} \cdots Z_{\underline{s}_{2k}} \rangle = \sum_{\text{all pairings}}^{\cup\{\alpha,\beta\} = \{1 \ldots 2k\}} \prod_{\alpha,\beta} \delta_{\underline{s}_\alpha, \underline{s}_\beta} + \mathcal{O}(L^{-1}),$$

while for odd number $\langle Z_{\underline{s}_1} Z_{\underline{s}_2} \cdots Z_{\underline{s}_{2k+1}} \rangle = \mathcal{O}(L^{-1})$, hence $Z_{\underline{s}}$ become independent *Gaussian* field variables in TL depending on a multi-index $\underline{s}$ of variable but finite length. Therefore, any TI *pseudolocal* (PL) observable A, having by definition [9] l^2-expansion in the basis $Z_{\underline{s}}$ (when $L = \infty$), namely, $A=\Sigma_s a_s Z_s$, $\langle A^2\rangle=\Sigma_s|a_s|^2<\infty$, possesses normal Gaussian moments $\langle A^{2k}\rangle=(2k-1)!!\langle A^2\rangle^k[1+\mathcal{O}(L^{-1})]$. Further, for a general TI PL observable A, its time average $\bar{A}$ is also TI PL, since it can be formally expanded in terms of Z_s due to construction of $\bar{A}$, and such expansion is l^2 since $\langle\bar{A}^2\rangle=\langle\bar{A}A\rangle=D_A<\langle A^2\rangle$ [12]. However, for a more general non-TI PL observable A, i.e., such that its *linear projection* to the space of TI observables $(1/L)\Sigma_{n=0}^{L-1}A|_{\vec{\sigma}_j\to\vec{\sigma}_{j+n}}$ is PL, one cannot generally show that $\bar{A}$ is TI PL although we believe that this is a typical situation, which we can prove in two cases.

(i) If the spectrum of propagator U is nondegenerate (for any finite L), then the matrix of $\bar{A}$ is diagonal in the eigenbasis of U and $\bar{A}$ is TI due to Bloch theorem.

(ii) If the system is integrable having a complete set of TI PL conservation laws $Q_n, n=1,2\ldots$ in the sense that $\{Q_n\}$ is a complete set of eigenvectors of the Heisenberg map $\hat{\mathcal{U}}A=U^\dagger AU$ for eigenvalue 1, then the time average is a projection $\bar{A}=\Sigma_n\langle Q_n A\rangle Q_n$ [assuming that $\langle Q_n Q_m\rangle=\delta_{nm}$] which is TI PL. This is the case for KI model studied below. Finally, assuming either (i), (ii), or simply TI PL perturbation A, we find that moments of time-average $\bar{A}$ are Gaussian $\langle\bar{A}^{2k}\rangle=(2k-1)!!D_A^k[1+\mathcal{O}(L^{-1})]$. Summing up the formula (8) produces Gaussian decay

$$F_{\rm ne}(t)=\exp[-(t/\tau_{\rm ne})^2/2], \quad (9)$$

for $t\gg t_{\rm ave}$, on a time scale (7), which can be computed in a typical integrable situation (ii) as shown bellow.

Few remarks on the case of finite dimension $\mathcal{N}<\infty$ are in order.

(1) $F(t)$ will then start fluctuating around zero with magnitude $F_{\rm fluct}\sim\mathcal{N}^{-1/2}$ for *very long times* $t>t^*(\mathcal{N})$ where the time scale $t^*(\mathcal{N})$ is determined from the condition $F(t^*)|_{\mathcal{N}=\infty}=F_{\rm fluct}$.

(2) $F(t)$ decays all the way down to $\sim\mathcal{N}^{-1/2}$ only for a *typical* or *random* initial state $|\psi\rangle$ with $\sim\mathcal{N}$ nonvanishing random components when expanded in the eigenbasis of U, or for an average over $|\psi\rangle$. If on the other hand one considers the initial state that, when expanded either in the eigenbasis of U or of U_δ, contains essentially only few, say m dominating components, e.g., the *regular* coherent state of Peres [3], then $F(t)$ is a quasiperiodic function with m small frequencies $\propto\delta$ and amplitudes $\sim 1/m^{-1/2}$.

(3) Even in asymptotically ergodic situation the correlation $C_A(t)$ has a plateau for finite $\mathcal{N}$, which can be estimated using a random matrix model for the observable A in the eigenbasis of the propagator U as $D_A\sim D_A^*(\mathcal{N}):=c_A/\mathcal{N}$ where c_A is some constant with respect to $\mathcal{N}$. The nonvanishing correlation plateau gives a dominant contribution to Eq. (4) resulting in a quadratic (or Gaussian) decay of $F(t)$ as soon as $\tau_{\rm e}>S_A|_{\mathcal{N}=\infty}/D_A^*$, i.e., when $\delta<\delta_{\rm p}(\mathcal{N}):=S_A^{-1}c_A^{1/2}\mathcal{N}^{-1/2}$. This *perturbative* regime of very small perturbation strength, existing for finite $\mathcal{N}$ only, is consistent with the first-order perturbation expansion of eigenstates of U_δ in terms of the eigenbasis of U [13].

Consider an example of KI model with the Hamiltonian

$$H_{\rm KI}(t)=\sum_{j=0}^{L-1}\{J_z\sigma_j^z\sigma_{j+1}^z+\delta_p(t)(h_x\sigma_j^x+h_z\sigma_j^z)\}, \quad (10)$$

where $\delta_p(t)=\Sigma_m\delta(t-mp)$, with a Floquet map $U=\exp(-iJ_z\Sigma_j\sigma_j^z\sigma_{j+1}^z)\exp[-i\Sigma_j(h_x\sigma_j^x+h_z\sigma_j^z)]$, where we take units such that $p=\hbar=1$, depending on a three independent parameters (J_z,h_x,h_z). KI is integrable for longitudinal $(h_x=0)$ and transverse $(h_z=0)$ fields [10], and has finite parameter regions of ergodic and nonergodic behaviors for a tilted field (see Fig. 1). The nontrivial integrability of a transverse kicking field, which somehow inherits the solvable dynamics of its well-known autonomous version [14], is quite remarkable since it was shown [10] that the Heisenberg dynamics can be calculated explicitly for observables that are bilinear in Fermi operators $c_j=(\sigma_j^y-i\sigma_j^z)\Pi_{j'}^{j'<j}\sigma_{j'}^x$, with time correlations decaying to the nonergodic stationary values as $|C_A(t)-D_A|\sim t^{-3/2}$ [10]. For D_A we find explicit expressions, the simplest,

$$D_{\sigma^x}=\frac{\max\{|\cos(2J_z)|,|\cos(2h_x)|\}-\cos^2(2h_x)}{\sin^2(2h_x)} \quad (11)$$

and $D_M=LD_{\sigma^x}$, for the component of spin σ_j^x, and the component of magnetization $M=\Sigma_j\sigma_j^x$, respectively.

In a general situation of nonintegrable KI we wish to test our theory by a numerical experiment. We consider a line in three-dimensional parameter space with fixed $J=1,h_x=1.4$ and varying h_z exhibiting all different types of dynamics: (a) $h_z=0$ *integrable*, (b) $h_z=0.4$ *intermediate* (nonintegrable and nonergodic), and (c) $h_z=1.4$ *ergodic* and *mixing*. In all cases we fix the operator $A=M$, which generates the perturbation of KI model with $h_x\to h_x+(h_x^2+h_z^2h\cot h)\delta/h^2+\mathcal{O}(\delta^2), h_z\to h_z+h_xh_z(1-h\cot h)\delta/h^2+\mathcal{O}(\delta^2)$, where $h=\sqrt{h_x^2+h_z^2}$, and vary L and δ. Since we want the perturbation strength to be size L independent, we scale it by fixing $\delta'=\delta\sqrt{L/L_0}$ where $L_0:=24$. Time evolution has been computed efficiently by iterating the factored Floquet map (in terms of one-spin and two-spin propagators—quantum gates), requiring $\propto L2^L$ computer operations per iteration per initial state. In integrable case (a) we confirm saturation of correlations to the theoretical value [10] $D_M=0.485\,126L$ [Fig. 1(a)], as well as Gaussian decay of fidelity (9) with time scale $\tau_{\rm ne}$ given by Eq. (7), which terminates at $t\approx t_{\rm ne}^*=\tau_{\rm ne}(\ln\mathcal{N})^{1/2}$ [Fig. 2(a)]. In nonintegrable (intermediate) case (b), we find persisting nonergodic and nonmixing behavior since rescaled correlation functions of typical observables $C_A(t)/\langle A^2\rangle$ relax on a short L-independent time scale to a nonvanishing value $D_A/\langle A^2\rangle$ and converge to TL very quickly with increasing size L [Fig. 1(b)], but as opposed to integrable case (a) the relaxation appears to be exponential $|C_M(t)-D_M|/L\sim\exp(-t/t_{\rm ave})$ with $t_{\rm ave}\approx 7.2$ [inset 1(b)]. Such behavior has been observed for other two components of the magnetization M^y,M^z and supports existence of intermediate dynamics observed previously in kicked t-V model [9]. In Fig. 2(b) we confirm Gaussian decay of $F(t)$ pre-

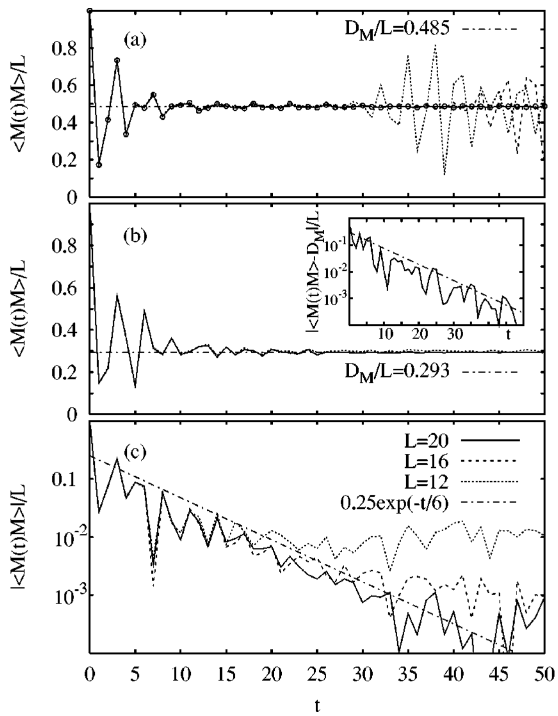

FIG. 1. Correlation decay for three cases of KI: (a) integrable $h_z=0$, (b) intermediate $h_z=0.4$, and (c) ergodic $h_z=1.4$, for different sizes $L=20,16,12$ [solid-dotted connected curves, almost indistinguishable in (a),(b)]. Circles (a) show exact $L=\infty$ result. Chain lines are theoretical/suggested asymptotics (see text).

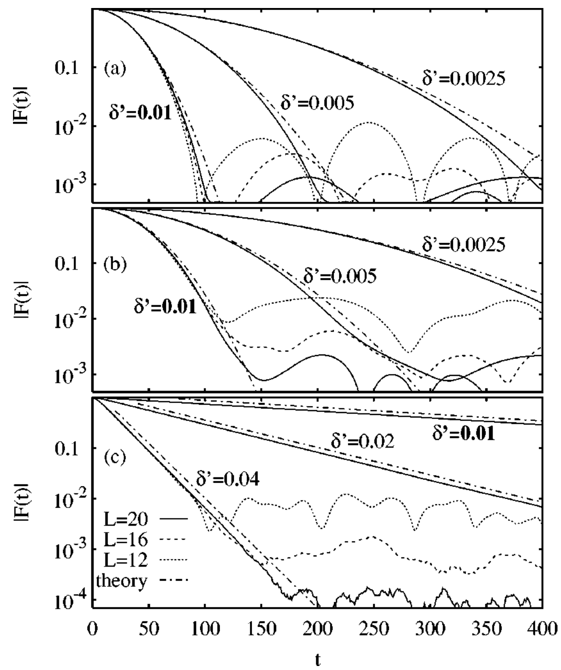

FIG. 2. Absolute fidelity $|F(t)|$ for three cases of KI: (a) integrable $h_z=0$, (b) intermediate $h_z=0.4$, and (c) ergodic $h_z=1.4$, for different sizes $L=20,16,12$ and different scaled perturbations δ'. Chain curves give theoretical predictions.

dicted Eq. (7) from numerically observed value of $D_M=0.293L$, again up to time $t^*_{\rm ne}(2^L)$. In ergodic case (c) we find fast decay of correlation functions fitting well to an exponential $|C_M(t)|/L \sim \exp(-t/t_{\rm mix})$, with $t_{\rm mix}\approx 6.0$. Consequently we find exponential decay of $F(t)$ of Eqs. (6) and (5) using $S_M=(1/2)\Sigma_t C_M(t)\approx 2.54L$, up to the saturation time $t^*_{\rm e}=(1/2)\tau_{\rm e}\ln\mathcal{N}$ [Fig. 2(c)].

In conclusion, we have presented a simple theory for the stability of quantum motion with respect to a static perturbation of the evolution operator in the limit of Hilbert space dimension $\mathcal{N}\to\infty$, characterized by the fidelity measuring the distance between time evolving states. The fidelity was expressed in terms of integrated time-correlation functions of the perturbing operator, showing that faster decay of correlations gives slower decay of fidelity, meaning that "chaotic" dynamics is more stable in Hilbert space than "regular" one (unless the state that one is looking at is simply related to the eigenstates of the system). In the two limiting cases of mixing and integrable (or more generally, nonergodic) dynamics we find, respectively, exponential and Gaussian decay. For example, our finding is predicted to have strong implication for the stability of quantum computation with respect to static imperfections (e.g., uncontrollable residual interaction among qubits) (see, e.g., Ref. [15] for a partial result in this direction). In other words, Eq. (5) is a version of the *fluctuation-dissipation* formula for the "dissipation coefficient" $1/\tau_{\rm e}$ of Eq. (6), which diverges in nonergodic regime. If the system has a well-defined classical limit then our formula (5) has a clear and simple classical limit $\hbar\to 0$ too, with an integrated classical autocorrelation function substituting the quantum one [16]. However, we note that our results on fidelity decay remain valid only if the limit $\delta\to 0$ is considered prior to the limit $\hbar\to 0$ as the two limits obviously do not commute. Furthermore, in systems having a chaotic classical limit, short-time behavior of the fidelity decay up to the Ehrenfest time $\sim -\ln\hbar$ will also depend on the structure of the initial state, which may range from a minimal uncertainty wave packet (coherent state) to a maximum entropy random state. These issues are dicussed in detail in Ref. [17]. We speculate that our finding is a manifestation of "the structural invariance" [18] of quantum chaotic dynamics. Although in this paper our theory has been demonstrated in a specific kicked many-body problem, namely, the quantum-kicked Ising spin-(1/2) chain, we should emphasize that it should be generally valid (within the time and perturbation scales depending on the Hilbert space dimension) and thus applicable to any unitary evolution, in particular, also to any experimentally interesting quantum dynamics.

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